Rob Pratt's method for Martin Gardner's problem on the minimum maximal no-three-queens-inline problem.

Integer variables:

x_{ij} for each point (i, j). This is an indicator variable that decides whether (i, j) is in the set.
y_k for each line. Use L_k to denote the set of points on the kth line. The variable y_k is such that if it is 1, then then L_k has exactly two points on it.

Objective: Minimize the total number of markers: $\sum x_{ij}$

Constraints:

$0 \leq v \leq 1$ for each variable

For each line k: $2\, y_k \leq \sum_{(i,j)\in L_k} x_{ij} \leq 2$

For each point (i, j): $x_{i\,j} + \sum_{(i,j)\in L_k} y_k \geq 1$

The first nontrivial constraint says that any line has at most two points and, if $y_k = 1$, then there are exactly two points on the kth line.

The second says that for any point, either it is in the set, or if not, then adding it to the set makes one of the lines have three points.

ILP works better if symmetry-breaking constraints are added. The next ones say that the number of points in the upper half of the board is at least as great as the number in the lower half; and similar for left and right and also the main diagonal.

$\sum_{j\leq\lfloor\frac{n}{2}\rfloor} x_{ij} \geq \sum_{j\geq\lceil\frac{n}{2}\rceil+1} x_{ij}$

$\sum_{i\leq\lfloor\frac{n}{2}\rfloor} x_{ij} \geq \sum_{i\geq\lceil\frac{n}{2}\rceil+1} x_{ij}$

$\sum_{i<j} x_{ij} \geq \sum_{i\geq>j} x_{ij}$

This can all be set up in *Mathematica* quite easily, using NMinimize as that is best for integer linear programming.